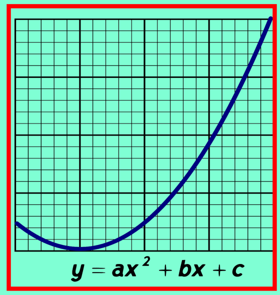
Math 125

Spring 2021

Lecture 28



Class QZ 20

1) Solve by Completing the square:

$$x^2 + 14x + 53 = 0$$

LC=1

even

$x^2 + 14x + 7^2 = -53 + 7^2$

$\frac{1}{2}(14) = 7$ $\quad (x+7)^2 = -4$

Use S.R.M.

$x + 7 = \pm\sqrt{-4}$

$x = -7 \pm 2i$

$\{-7 \pm 2i\}$

2) Discuss the type of Solutions for

$$3x^2 - 4x + 10 = 0$$

$$ax^2 + bx + c = 0$$

$a = 3$

$b = -4$, $c = 10$

$b^2 - 4ac = (-4)^2 - 4(3)(10) = 16 - 120 = -104$

$b^2 - 4ac < 0 \Rightarrow$ Two Imaginary Solutions

Solve $x^4 - 5x^2 - 36 = 0$ by making Proper Subs.

Notice $x^4 = (x^2)^2$

$x^4 - 5x^2 - 36 = 0 \Rightarrow (x^2)^2 - 5x^2 - 36 = 0$

Let $u = x^2 \Rightarrow u^2 - 5u - 36 = 0$

$(u-9)(u+4) = 0$

$u - 9 = 0 \qquad u + 4 = 0$

$u = 9 \qquad u = -4$

$u = 9$

$x^2 = 9$

$x = \pm\sqrt{9}$

$\boxed{x = \pm 3}$

$u = -4$

$x^2 = -4$

$x = \pm\sqrt{-4}$

$\boxed{x = \pm 2i}$

$\boxed{\{\pm 3, \pm 2i\}}$

Solve $2x^{2/3} - 3x^{1/3} - 5 = 0$ by letting

$u = x^{1/3}$.

$u^2 = (x^{1/3})^2 = x^{2/3}$

$2x^{2/3} - 3x^{1/3} - 5 = 0 \Rightarrow 2u^2 - 3u - 5 = 0$

$(2u - 5)(u + 1) = 0$

$2u - 5 = 0 \qquad u + 1 = 0$

$u = \frac{5}{2} \qquad u = -1$

$u = \frac{5}{2}$

$x^{1/3} = \frac{5}{2}$

$\sqrt[3]{x} = \frac{5}{2}$

$x = \left(\frac{5}{2}\right)^3$

$\boxed{x = \frac{125}{8}}$

$u = -1$

$x^{1/3} = -1$

$\sqrt[3]{x} = -1$

$x = (-1)^3$

$\boxed{x = -1}$

$\left\{-1, \frac{125}{8}\right\}$

Find a quadratic equation in $ax^2+bx+c=0$ form with $2 \pm 3i$ as Solutions.

$x = 2 + 3i$ $\qquad$ $x = 2 - 3i$

$x - 2 - 3i = 0$ $\qquad$ $x - 2 + 3i = 0$

$$\underbrace{(x-2 \;-3i)(x-2\;+3i)}_{\text{Conjugates}} = 0$$

Conjugates $\longrightarrow (a-b)(a+b) = a^2 - b^2$

$(x-2)(x-2)$

$$(x-2)^2 - (3i)^2 = 0$$

$$x^2 - 4x + 4 - 9i^2 = 0$$

$$\longrightarrow x^2 - 4x + 4 - 9(-1) = 0$$

$$\boxed{x^2 - 4x + 13 = 0}$$

Graph $\qquad f(x) = (x+3)^2 + 2$

Hint: $f(x) = a(x-h)^2 + K$

$a = 1 \quad h = -3 \quad K = 2$

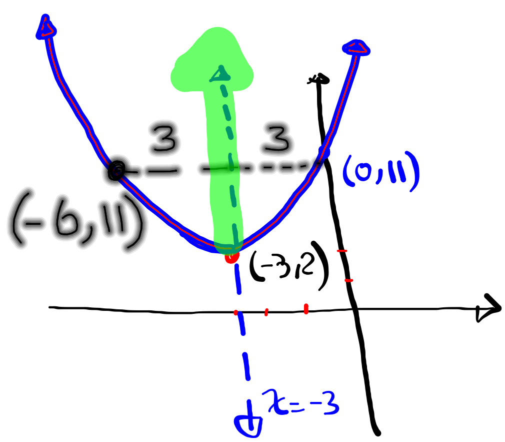
1) Vertex $(h, K) = (-3, 2)$

2) Direction opens upward
$a > 0$

3) A.O.S. $x = h \qquad x = -3$

4) Intercepts Y-Int $(0, 11)$
No x-Int.

5) Domain & Range $\quad$ Domain: $(-\infty, \infty)$ $\quad$ Range: $[2, \infty)$



Graph $f(x)=-\frac{1}{2}x^2+2x$

Hint: $f(x)=ax^2+bx+c$

$h=\frac{-b}{2a}$, $k=f(h)$

1) Vertex $(2,2)$

$a=\frac{-1}{2}$ $b=2$ $c=0$

2) Direction opens downward $a<0$

$h=\frac{-b}{2a}=\frac{-2}{2(\frac{-1}{2})}=\frac{-2}{-1}=2$

3) A.O.S. $x=h$ $x=2$

$k=f(2)=\frac{-1}{2}(2)^2+2(2)$

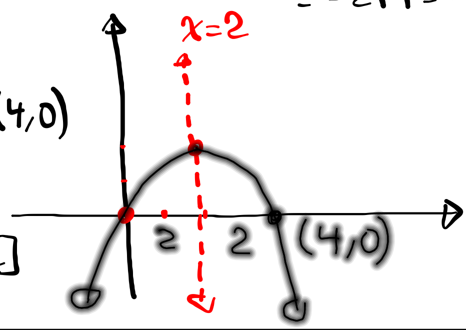
$=-2+4=2$

4) Intercepts Y-Int $(0,0)$

x-Int $(0,0)$ & $(4,0)$

5) Domain & Range

Domain: $(-\infty,\infty)$, Range: $(-\infty,2]$



Graph $x=-(y-3)^2+2$

Hint: $x=a(y-k)^2+h$

$a=-1$

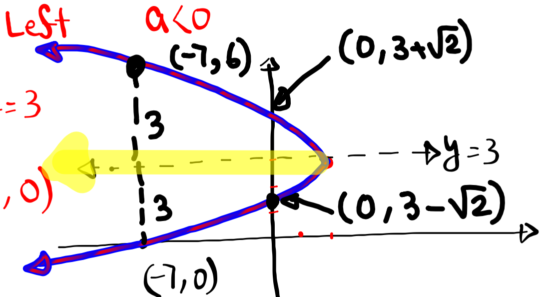
$k=3$ $h=2$

1) Vertex $(h,k)=(2,3)$

2) Direction opens Left $a<0$

3) A.O.S. $y=K$ $y=3$

4) Ints x-Int $(-7,0)$



5) Domain & Range

Y-Int $\rightarrow x=0$

$-(y-3)^2+2=0$

$-(y-3)^2=-2$

$(y-3)^2=2$

$y-3=\pm\sqrt{2}$

$y=3+\sqrt{2}$

$y=3-\sqrt{2}$

Domain: $(-\infty,2]$

Range: $(-\infty,\infty)$

Graph $x = y^2 - 6y + 9$

1) Vertex $(0,3)$

2) Direction opens Right $a > 0$

3) A.O.S. $y = k$ $y = 3$

4) Ints x-Int $(9,0)$
y-Int $(0,3)$

5) Domain & Range

Domain: $[0, \infty)$

Range: $(-\infty, \infty)$

Hint: $x = ay^2 + by + c$

$k = \frac{-b}{2a}$

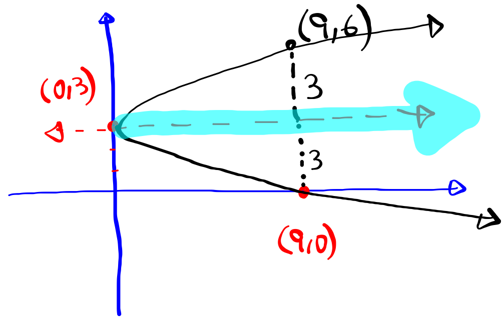
h = plug in k for y.

A.O.S. $y = k$

$a = 1$ $b = -6$ $c = 9$

$k = \frac{-b}{2a} = \frac{-(-6)}{2(1)} = \frac{6}{2} = 3$

$h = 3^2 - 6(3) + 9 = 0$



Class QZ 21

1) Find a quadratic equation in the form of $ax^2 + bx + c = 0$ with Solution $-4 \pm 2i$.

$x = -4 + 2i$ $x = -4 - 2i$

$x + 4 - 2i = 0$ $x + 4 + 2i = 0$

$(x + 4 - 2i)(x + 4 + 2i) = 0$

$(x+4)^2 - (2i)^2 = 0$

$x^2 + 8x + 16 - 4i^2 = 0$

$x^2 + 8x + 16 - 4(-1) = 0$

$\boxed{x^2 + 8x + 20 = 0}$

2) What is vertex of $f(x) = -2x^2 - 8x$?

$f(x) = ax^2 + bx + c$

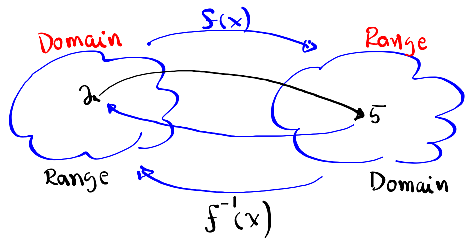
$a = -2$ $b = -8$ $c = 0$

$h = \frac{-b}{2a} = \frac{-(-8)}{2(-2)} = \frac{8}{-4} = -2$

$k = f(h) = -2(-2)^2 - 8(-2) = -8 + 16 = 8$

Vertex $(-2, 8)$

March 29, 2021



$f(2)=5$

$f^{-1}(5)=2$

"f-inverse of x"

How to find $f^{-1}(x)$:

1) Replace $f(x)$ with y.
2) Switch x & y.
3) Solve for y.
4) Replace y with $f^{-1}(x)$.

Ex: $f(x)=3x-2$

$y=3x-2$

$x=3y-2$

$x+2=3y$

$y=\frac{x+2}{3}$

$\boxed{f^{-1}(x)=\frac{x+2}{3}}$

Check

$f(2)=3(2)-2=6-2=4$

$f^{-1}(4)=\frac{4+2}{3}=\frac{6}{3}=2$

$F(2)=4$

$f^{-1}(4)=2$

Given $f(x)=2x+5$

1) Find $f(-1)=2(-1)+5=-2+5=3$ $\boxed{f(-1)=3}$

2) Find $f^{-1}(x)$

$f(x)=2x+5$

$y=2x+5$

$x=2y+5$

$x-5=2y$

$\frac{x-5}{2}=y$

$\boxed{f^{-1}(x)=\frac{x-5}{2}}$

3) Verify $f^{-1}(x)$ by using result of Part 1.

$f^{-1}(3)=\frac{3-5}{2}=\frac{-2}{2}=-1$

$F^{-1}(3)=-1$

$F(-1)=3$

$f(x) = \sqrt{x-2}$

1) Find $f(2)$ and $f(11)$

$f(2) = \sqrt{2-2} = \sqrt{0} = \boxed{0}$ $\quad F(11) = \sqrt{11-2} = \sqrt{9} = 3$

$\boxed{f(2)=0}$ $\quad \boxed{f(11)=3}$

2) Find $f^{-1}(x)$

$F(x) = \sqrt{x-2}$

$y = \sqrt{x-2}$

$x = \sqrt{y-2}$

$x^2 = y-2$

$\boxed{y = x^2+2}$

$\boxed{F^{-1}(x) = x^2+2}$

3) verify $f^{-1}(x)$ by results obtained in part 1.

$F^{-1}(0) = 0^2+2 = 0+2 = 2 \Rightarrow F^{-1}(0) = 2$

$F(2) = 0$

$f^{-1}(3) = 3^2+2 = 9+2 = 11 \Rightarrow F^{-1}(3) = 11$

$f(11) = 3$

$f(x) = \sqrt{x+1}$

1) Find $f(-1)$ & $f(15)$

$f(-1) = \sqrt{-1+1} = \sqrt{0} = 0$

$f(15) = \sqrt{15+1} = \sqrt{16} = 4$

$\boxed{\begin{array}{l} F(-1)=0 \\ f(15)=4 \end{array}}$

$F^{-1}(0) = -1$ ✓

$F^{-1}(4) = 15$ ✓

2) Find $f^{-1}(x)$

$y = \sqrt{x+1}$

$x = \sqrt{y+1}$

$x^2 = y+1$

$x^2-1 = y$

$\boxed{f^{-1}(x) = x^2-1}$

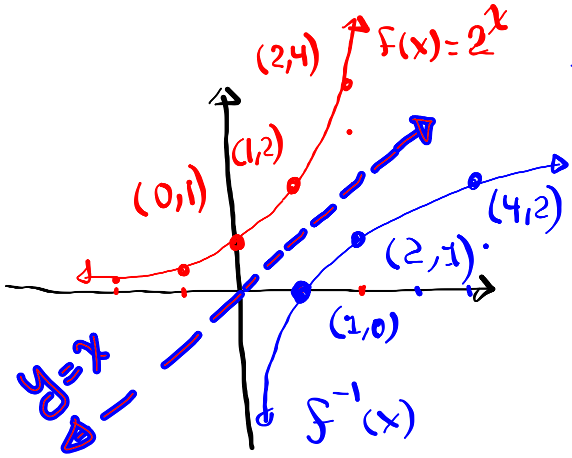
3) verify $f^{-1}(x)$ by results obtained in part 1.

$F^{-1}(0) = 0^2-1 = 0-1 = -1$

$F^{-1}(4) = 4^2-1 = 16-1 = 15$

Exponential Function

$f(x) = b^x$, $b > 0$, $b \neq 1$ $\quad F(x) = 2^x$



x	$f(x)$
0	$2^0 = 1$
1	$2^1 = 2$
2	$2^2 = 4$

x	$f(x)$
-1	$2^{-1} = \frac{1}{2^1} = \frac{1}{2}$
-2	$2^{-2} = \frac{1}{2^2} = \frac{1}{4}$

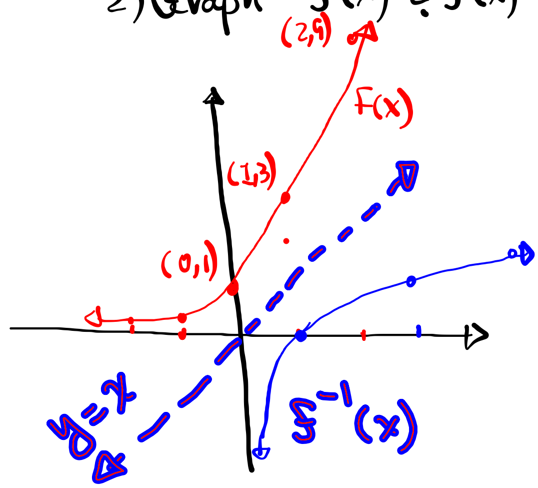
$f(x) = 3^x$

1) Complete the tables below

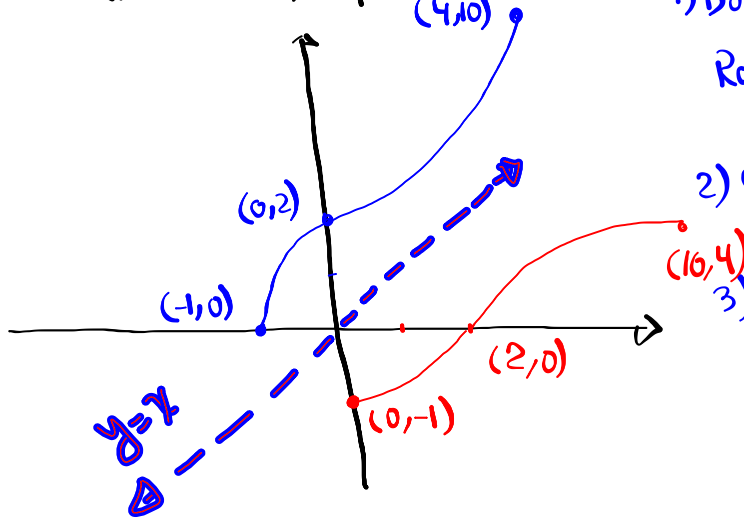
x	$f(x)$
0	$3^0 = 1$
1	$3^1 = 3$
2	$3^2 = 9$

x	$f(x)$
-1	$3^{-1} = \frac{1}{3}$
-2	$3^{-2} = \frac{1}{9}$

2) Graph $f(x)$ & $f^{-1}(x)$



Consider the graph below



1) Domain: [-1,4]

Range: [0,10]

2) graph its inverse

3) Domain: [0,10]

Range: [-1,4]

for the inverse.